Bonus problem (3 pages, additional 5 points)

1. Problem 1

For each conservative matrix V, we have:



This means that V is skew-symmetric.

On the other hand, we can conclude that not all skew-symmetric matrices are conservative, here is an example:





So not all skew-symmetric matrices are conservative.

We define a skew-symmetric matrix of the form V (i, j) = αi − αj is called additive.

We can conclude that every additive matrix is conservative:





So V is conservative.

We can also conclude that every conservative matrix V is additive:







Finally, the space of 6×6 conservative matrices had dimension 5 and this is because the matrix is constructed by {V (1, 2), V (1, 3), V (1, 4), V (1, 5), V (1, 6)}. All the remaining parts can be calculated as a linear combination of these five values: V(u,j) = V(u,1)+V(1,j) = −V(1,u)+V(1,j). So this will be the least components.

1. Problem 2

For a single k × k table, obtain an expression for the least-squares estimate of α. Use this formula to compute αˆ for each of the three electrolytes. Explain why (α1, · · · , α5) and (α1, · · · , α5) + (c, c, c, c, c) are equivalent as parameter points in the model.

We can set this up as a typical least squares problem by transforming the k × k matrix Y into a

k2 × 1 vector y. Then, if R is the usual matrix of 1s and 0s for the row as a categorical variable, and C is the same for the column, our model matrix is X = R − C. This gives us E(y) = Xα. After omitting one of the columns of X due to collinearity, we can then produce the normal least squares estimate α = (X′X)−1X′y. αi −αj = (αi +c)−(αj +c), so {α1, α2, α3, α4, α5} and {α1 +c, α2 +c, α3 +c, α4 +c, α5 +c} are equivalent parameters, demonstrating that one parameter needs to be omitted.

When we preform this procedure on the data, we get these predictions for the α by electrolyte

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Zn | *Fe* | *Pb* | *Cu* |
| O | -0.3908 | -0.8408 | -0.9138 | -1.2426 |
| A | -0.3962 | -0.8690 | -0.9624 | -1.3284 |
| K | -0.4398 | -0.8850 | -0.9566 | -1.3106 |

Table Alpha for the Three Electrolytes

1. Problem 3

These two models are one just including the α, and one where the α are interacted with electrolyte. Running these two models and comparing them with ANOVA, we get:

By the interaction, we don’t see quite enough of an improvement to push past the .05 p-value threshold, though it is a borderline case.

1. Problem 4

Our data set only has factor covariates, so no monotone transformation will have an effect on the predictions, and non-monotone transformation wouldn’t make sense. If there was evidence of a lack of normality or non-constant variance, a transformation might improve our statistical inference, but checking various residual plots leaves little indication that this is necessary.